DIRECT VARIATION and PROPORTIONALITY
Day One

LESSON TITLE: Direct Variation

Overview

Grade: 8
Course: Math 8
Time Allotment: one class period

Common Core State Standards - Mathematics 8:

- 8.EE.6
  - **Supporting Standards:**
    - **8.G.3, 8.G.4, 7.RP.1-3**

Standards of Mathematical Practice:

- Standard 7
  - Supporting Practices:
    - 1,6,and 8

Goals:

Students will use similar triangles and equivalent and ratios to explain why slope is the same between two distinct points on a line of form y=mx+b

Objectives:

- Students will know that a series of points create a line using similar triangles from two data points within that line.
- Students identify slope under direct variation and know the line's equation of the form y=mx

Materials:

Chart paper, graph paper, markers, Handouts A, B, and C, and pre-made graph **on chart paper or and tech device.**

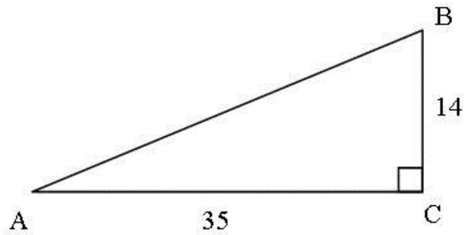
Technology implemented:

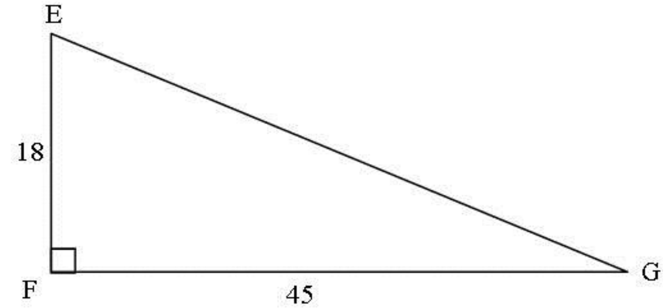
Smart-board, and or doc camera if needed.

Opening Procedure (7-10 minutes)

1. **Warm up(5-7 minutes)**
   When students walk in, have them get situated and then proceed with giving them the warm up on chart paper, SmartBoard, and or doc camera.

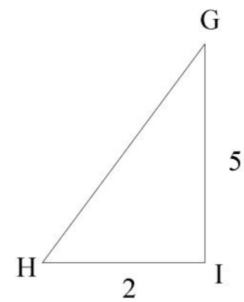
1A:





Are the above triangles similar? Explain using numbers and explanations Whether they are or aren't.

1B: Would ΔGHI also be similar to ΔABC and ΔDEF? Why or why not?



Teacher Note: Remind students of similar triangles and equivalent triangles and equivalent rations. We assume 8.G3,4 have already been taught. This is a review of those standards.

2. Find the rule of the given table and put it into an algebraic sentence.

X	Y
7	21
20	60
9	27
x	

Teacher Note: The above problem is for a sneak preview of prior knowledge to help later in the lesson

2. **Head Problems:(2-3)**
Optional to process warm-up after; sometimes it is nice to "pull students out" of warm up with a head problem and then go back to warm up to process.

a. Take the number 6, double it , take a third of that, now take half of that. Show me on your fingers the number you have. (You should end up with two)

Purpose: avoid trouble with ratio relationships.

Teacher Note: Please replay the head problem with the students. Begin by asking questions such as, "I will take a quiet hand if someone can tell me what number I started with. Can someone please tell me what I did next? Everybody what is 6 doubled? Whisper to your neighbor what I did next. Can someone tell me what your partner said?" Continue this until you are done replaying. Make sure that you ask, "What is the math for thirds, halves, and maybe even fourths?"

<u>The Lesson:</u>

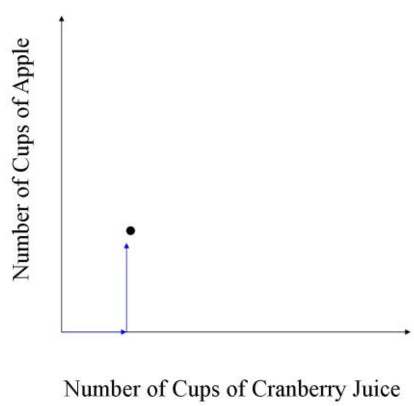
<table>
<tr><th>Teacher Do</th><th>Students Do</th></tr>
<tr><td>Chunk #1 Purpose: To have students graph, slope and tracking the points to show that the points are continuous (3-5 minutes)</td><td></td></tr>
<tr><td>***Make sure you leave up all work from this section and from warm-up. This will be great reference for the future.

1. The teacher starts off with the posing scenario:

"We're having a party and you're making your famous cran-apple punch. It is from combining cranberry juice and apple juice. My famous recipe calls for every one cup of cranberry juice, you add 5 cups of apple juice."

After you pose the scenario, reference the blank cranberry-apple juice graph. Ask students how to plot this data point from the given ratio of the punch. Make sure they have you start at the origin! Have students help you as you motion going over and up. It is helpful to have the students tell you to "stop" as you go over on the x-axis and then "stop" when you get to the point on the y-axis.

Do NOT draw the arrows at this time. It may be distracting to the students. The objective at this time is to have the students generate points and create a continuous line graph.



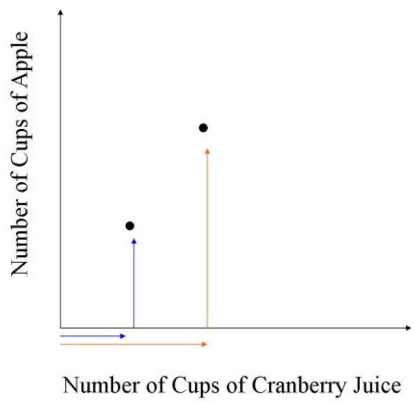
2. Once this first point is plotted, give next scenario and place new point one same graph as above graph:</td><td>Students help with quiet hands to draw this point of (1,5). Maybe students show the movement of the over-up to plot the point as the teacher does with markers with their fingers.

Students will repeat the same</td></tr>
</table>

"What if you made a bigger batch using 2 cups of cranberry juice, how much apple would you need to keep the famous punch tasting the same?"

Ask for predictions, then go to the math to prove the number of cups of apple juice.

Once again, motion the movement to the right and up when plotting the point. Do NOT draw the arrows at this time. It represents the movement of your hand and pen.



process as above by using their fingers to emulate the movement of points. You could have students predict the pattern of the points act on the line in this relationship.

3. Get students to help you start organizing the plots on the graph into the table. Perhaps ask the students, "Is there a way to keep track of our data points in an organized manner?" The students would then encourage you to use a table.

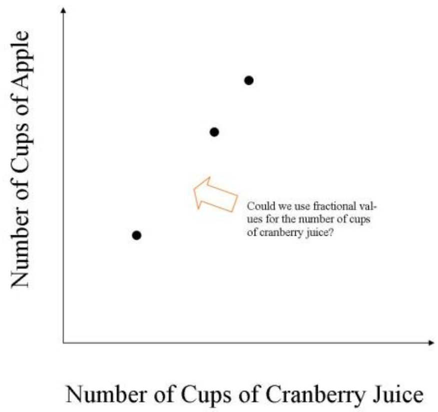
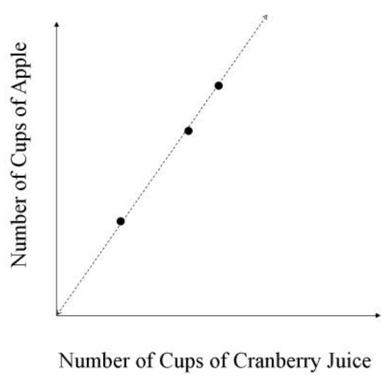
Students suggest using a table and generate values that follow the recipe

Ex:

# of cups of Cranberry Juice	# of cups of apple juice
1	5
2	10

Get students to tell you different entries for the table less than 10 cups of cranberry juice. Use manageable numbers such as whole numbers. As the students share points, plot them on the graph.

Once several more points are plotted, ask students, "Could we use ½ cup of cranberry juice?" Use similar questions to solidify that this is a continuous graph with values in

between the whole numbers, not just a few points.  	
Now plot a point that does NOT follow the recipe, i.e. does not appear on the line. For example, plot (½, ½) or (1, 2). Say to the students, "Okay, so this point works. It's fractional, right?" as a deliberate mistake. Once students reason that the coordinate does not fit the relationship and that it is a continuous one, erase the point and draw the line.  	Students should be able to reason that these coordinates do not follow the recipe.
Chunk #2 Purpose: To have students recognize similar triangles exist between any two points on the line, i.e. that the rate of change between any two points is proportional (15-20 minutes)	
4. Pitch a new scenario to the students: "One of your friends likes your punch recipe, but prefers her drinks to be a bit more tart. How can that be done?" May ask students to talk to a partner first to brainstorm, then ask students to share thoughts. After students share reasoning with each other whole class, ask: "After lots of experimentation, your friend decided she likes 3 cups of cranberry juice for every 1 cup of apple juice the best. Hmm…since we have a new scenario, how many people agree that	**Possible student responses:** **(1) Add fewer cups of apple juice for every cup of cranberry** **(2)Add more cups of cranberry for every 5 cups of apple juice** **Students agree**

we need a new graph?"

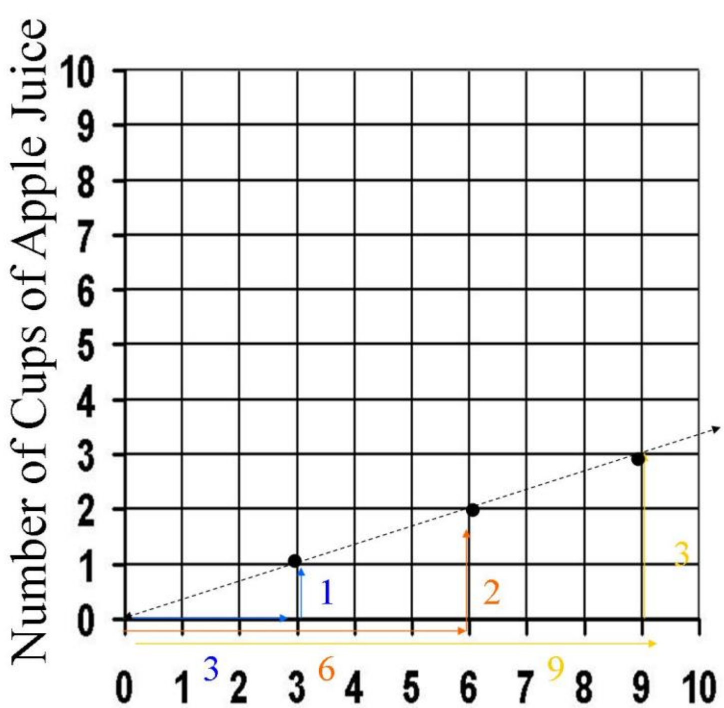
"With our new recipe, show me on your fingers how many cups of cranberry juice we use? Show me how many apple? How many people think we can plot this quickly on the graph?"

Ask students to help you plot (3,1) by using the same method previously. HOWEVER, include the directional arrows and their lengths on your graph (will be used to show the similar triangles that appear to prove that the slope between any two points on a line is the same).

Ask the students the following types of questions to help graph the data and vary the mode of response:

(1) How many cups of apple juice are needed for 9 cups of cranberry juice?
(2) Let's make a batch that is twice as big as the original batch (i.e. 3:1). What do we add?
(3) Can we have ¾ cup, 1 ½ cups, etc. of cranberry juice? (Prove that the relationship is still continuous and that a line can be drawn to connect these points)
(4) Are we sure this is a straight line? How do we know for sure?
(5) Does this point follow the recipe? Why or not? If not, is it more "applier" or "cranberrier"?
(6) What does the x-value and y-value represent?
(7) What kind of line is this? (Choral response: CONTINUOUS)

After working with the students collaboratively, your graph should look similar to the one below.



by raising hands, showing on their fingers

5. Pass out Student Handout A. Ask them to draw two new right triangles connecting any two points on the line (one should NOT use the origin as a point). Teacher circulates the room to monitor student progress. Teacher can also do some "thinking-aloud" to sprinkle hints (1:3, 3:9…interesting lengths). Pick some students to draw their triangles on the board. It would be nice to choose a student who drew an "upside-down" triangles (as it emphasizes the relationship between similarity and transformations).	**Encourage students to work in groups or partners.** **Since students will struggle to see other slope triangles. May be helpful to revisit the slope triangles already drawn on graph on the board; point to the two points that are creating the right triangle and show how to draw the corresponding legs.**
Once several slope triangles have been drawn, ask, "What do we notice about our new triangles?"	**Students responds, "They are similar triangles."**
Once students respond, ask, "What does similar mean again? Talk to your partner."	**Students remind one another, using Daily Two as a possible example**
Ask students to share their thoughts on similarity, i.e. what makes these triangles similar. Choose any two triangles drawn and the graph. Ask, "Oh…so are these two similar? And these two?" Do enough where students realize ALL combinations of triangles created by two points on the line are similar.	
"Oh, okay. How many people agree that the slope between any two points on a line is the same, right?" (Purposefully using the definition of slope in context for the first time instead of defining it with the dictionary definition).	**Student raise hands to agree**
Chunk #3 Purpose: Recognizing that m in the equation y = mx represents the proportion, i.e. rate of change or slope (10-12 minutes)	
6. Refer again to Student Handout A. Ask students to: (1) Determine the recipe and (2) Make three entires in the t-	**Students work in partners or**

<table>
<tr>
<td>chart that follow the recipe. Teacher circulates room, checking in with each group to monitor progress.

After students have made sufficient progress, teacher pulls students out of groups to share out. Gather four to five coordinate pairs that fit the recipe and record them in a t-chart on the board.

Ask students, “If we had x cups of cranberry juice, how many cups of apple juice would we need to make sure it tastes the same?”</td>
<td>groups to complete task

Students share out solutions

Possible students responses:

<table>
<tr><th></th><th>Cran</th><th>App</th></tr>
<tr><td>A</td><td>2x</td><td>x</td></tr>
<tr><td>B</td><td>x</td><td>x÷2</td></tr>
<tr><td>C</td><td>x</td><td>2x</td></tr>
<tr><td>D</td><td>x</td><td>½x</td></tr>
</table>

A & B: Correct, but not helpful for the slope triangle

C: Question to help student- “What do we have more of? Cranberry or apple juice?”

D: Desired response. Students can see where vertical change and horizontal change appear in the ratio.</td>
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<td>Once students have the correct algebraic expression, circle/highlight the ½ in the equation and ask, “How is this number related to the recipe?”</td>
<td>Students do NOT respond…simply planting a seed</td>
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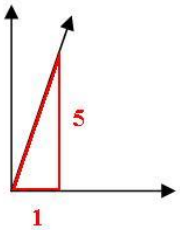
<tr>
<td>7. Revisit old two t-charts (5:1 and 1:3 recipes). Ask students, “How many cups of apple juice are needed for x cups of cranberry juice for each recipe?” This creates two more rules, for a total of 3 equations. Scaffold by adding more numerical examples of each recipe and/or write the operations showing how the numerical answers are found.</td>
<td>Students verbalize answers

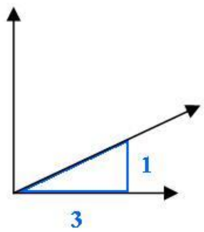
Possible student response: Some may recognize that coordinates themselves are</td>
</tr>
</table>

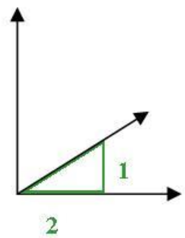
# cups of cranberry	# cups of apple
1	$5 = 1 \cdot 5$
2	$2 = 2 \cdot 5$
.	.
.	.
.	
x	$x = x \cdot 5$ or $5 \cdot x$

# cups of cranberry	# cups of apple
3	$1 = 3 \div 3$ or $\frac{3}{3}$
6	$2 = 6 \div 3$ or $\frac{6}{3}$
.	.
.	.
.	.
x	$x = x \div 3$ or $\frac{x}{3}$ or $\frac{1}{3}x$

# cups of cranberry	# cups of apple
2	$1 = 2 \div 2$ or $\frac{2}{2}$
4	$2 = 4 \div 2$ or $\frac{4}{2}$
.	.
.	.
.	.
x	$x = x \div 2$ or $\frac{x}{2}$ or $\frac{1}{2}x$







the same as the coefficient in the equation. This is accurate for now (definition of unit rate). So may tell the students we will want to keep an eye on this relationship to see if it is ALWAYS true for any linear relationship.

Guide the students through a discussion summarizing all the patterns we have seen today.

Students should articulate:

A. The coefficient is (1) recipe, (2) slope/rate, and (3) the ratio of similar triangles.

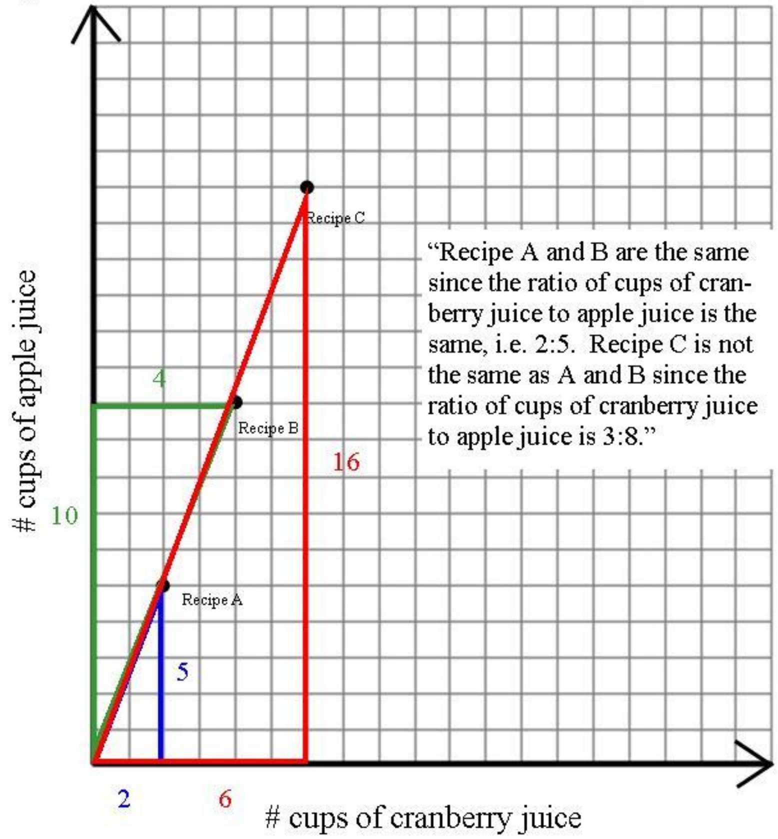
B. All three of these graphs pass through the origin (since these relationships are direct variations)

Closure/Homework:

Have students complete the activity on the backside of Student Handout A. If there is not enough time to complete in class, could be assigned as homework or used as a Daily Two the next day.

See images illustrating possible student answers.

1.



2.

# cups of cranberry	# cups of apple juice
7	21
20	60
9	27
x	$3x$

$\frac{21}{7}$ $\frac{60}{20}$ $\frac{27}{9}$

Response A: For any number of cups of cranberry juice, there is 3 times as many cups of apple juice. On a graph, these points would create a series of similar triangles whose ratio of corresponding sides would always be 3:1. Therefore, all the recipes

Response B: Each of these ratios are the same, i.e. 3:1. On the graph, these are the dimensions of the triangles formed with the coordinate and the origin.